

The definition of the differential is given in a box at the bottom of page 150:

$$dy = f'(x)dx$$

To understand what that means, look at Figure 4-64 on page 151. Notice that there are *two* triangles with the same base $dx = \Delta x$.

The taller triangle has altitude Δy . The shorter triangle has altitude dy .

Since $\frac{dy}{dx}$ is the slope of the tangent line, the altitude of the shorter triangle is the slope times the base. In other words, $dy = \frac{dy}{dx} dx = f'(x) dx = f'(x) \Delta x$.

What is the significance of this? **If we know Δx we can approximate Δy by finding dy .**

Examples 1-3 at the top of page 151 don't really show us anything new. They just have you find dy rather than $\frac{dy}{dx}$. However, notice that all you do is apply the same rules for differentiation.

Here's an example of a more practical problem. Return to Example 3 on page 151, the balloon.

Suppose we want to estimate how much the volume of the balloon increases when the radius increases from 3.00 ft to 3.01 ft.

$$V = \frac{4}{3}\pi r^3 \quad \text{so} \quad \frac{dV}{dr} = 4\pi r^2 \quad \text{as we saw before.}$$

This means that $dV = 4\pi r^2 dr$.

In this situation $r = 3.00$ ft and $dr = .01$ ft.

When we substitute we find that $dV = 0.36\pi \approx 1.13 \text{ ft}^3$.

In Example 4, Δx is a small number, 0.01. This example shows that when dy is found by taking the derivative and Δy is found by the delta process, the results are almost the same.

Skip Example 5.

Examples 6 and 7 show how to apply the differential to find approximate measurement error. Study these examples and be sure you understand the terms **absolute error** and **relative error**.

This chart summarizes the results:

Measurement	Absolute Error	Relative Error
Edge	$de = 0.020 \text{ cm}$	$de/e = 0.52\%$
Volume	$dV = 0.89 \text{ cm}^3$	$dV/V = 1.6\%$

Notice that $dV/V = 3de/e$. This is no accident! Start with the original differential equation:

$$dV = 3e^2 de \text{ and divide both sides by } V = e^3.$$

Skip Example 8.

Homework: page 154: 5, 7, 9, 25, 29, 31

pages 156-157: 19, 21, 23, 43, 47, 49, 63 (includes review items)